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Economic Statistical Design of a Three-Level Control Chart with VSI Scheme

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Abstract:

Traditionally, the statistical quality control techniques utilize either attributes or variables product quality measure. Recently, some methods, such as a three-level control chart, have been developed for monitoring multi attribute processes. Control chart usually has three design parameters: the sample size (n), the sampling interval (h) and the control limit coefficient (k).The design parameters of the control chart are generally specified according to statistical or/and economic criteria. The variable sampling interval (VSI) control scheme has been shown to provide an increase to the detecting efficiency of the control chart with a fixed sampling rate (FRS). In this paper, a method is proposed to conduct the economic-statistical design for a variable sampling interval of the three-level control charts. We use the cost model developed by Costa and Rahim and optimize this model by a genetic algorithm approach. We compare the expected cost per unit time of the VSI and FRS 3-level control charts. Results indicate that the proposed chart has improved performance.

Keywords: Three-level control chart; Variable sampling interval control scheme, Economic- statistical design, Genetic algorithm.

Mathematics Subject Classification (2010): 99X99, 99X99.

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1. Introduction

Control charts are widely applied to monitor and improve the quality and productivity of industrial processes and service operations. Control chart usually has three design parameters: the sample size (n), the sampling interval (h) and the control limit coefficient (k). Standard Shewhart charts are used with fixed design parameters, which called fixed sampling rate (FRS). Several modifications adopting a variable sampling interval (VSI) in the control chart have been suggested to improve the traditional FRS policy. They are evaluated in the control quality literature, for example: Cui and Reynold (1988), Reynold et al. (1988), Runger and Pignatiello (1991), Reynold (1996), Aparisi and Haro (2001), Faraz et al. (2010), Kazemzadeh et al. (2012), and Yang (2013). These researches have shown that using the VSI control chart is substantially quicker than using the FRS control chart in detecting small or moderate shifts in the process.

The design parameters of the control chart are generally specified according to statistical or/and economic criteria. In a statistically designed control chart, the design parameters are chosen, such that the chart meets some statistical performance requirements, while the minimization of the net sum of all costs involved yields an economical design. For an economic-statistical design (ESD), the parameters are chosen to minimize the costs subject to some constraints on the statistical performance. The economic design was introduced by Duncan (1956). Thereafter some of the researchers have studied the economic design of the FRS control charts (see, for example, Montgomery and Heikes (1976), Lorenzen and Vance (1986) and Costa and Rahim (2001)). Das et al. (1997), Bai and Lee (1998), Bai and Lee (2002) investigated the economic performance of a VSI chart with a single assignable cause model. Yu and Hou (2006) considered the economic design of a VSI chart with multiple assignable causes. Chen (2004), Li et al. (2009), and Chen and Yeh (2010) developed the economic design of VSI control charts for non-normal observations.

Economic-statistical design was proposed by Saniga (1989) and extended by several authors such as Prabhu et al. (1997), Magalhaes et al. (2002), Chen (2003), Niaki et al. (2010) Chen and Yeh (2011), Faraz and Saniga (2011), Yeong et al. (2013).

Shewhart control charts often are constructed for a variable or an attribute quality characteristic of interest. However, a quality characteristic may be measured using three or more discrete levels in some situations. Cassady and Nachlas (2003) proposed a three-level classification scheme thet classifies the quality of products into of the three categories called “conforming”, “marginal” or “nonconforming". The Shewhart control charts based on the three-level classification scheme have been discussed by Cassady and Nachlas (2006). Three-level Shewhart control charts are a quite useful tool for detecting process shifts for three-level products, and they are easy to operate for practitioners. However, the three-level Shewhart control chart is insensitive for detecting small process shifts. Tsai and Yen (2011) developed exponentially weighted moving average control charts for multinomial data with a three-level classification scheme. Pourtaheri (2017) presented the statistical design of variable parameters, three-level control charts.

In this paper, we develop a control chart based on a VSI control scheme to enhance the ability to detect small shifts in a three-level multinomial process. In addition, we use the economic-statistical model to consider both economic and statistical criteria. This paper is organized as follows: Section 2 describes the VSI three-level Shewhart control charts. In Section 3, the performance of the proposed chart is achieved in terms of $AATS$ measure using a Markov chain approach. The cost model proposed by Costa and Rahim (2001) is presented in section 4. A numerical comparison between VSI and FRS for statistical and economical criteria is shown in section 5, and the final section provides some concluding remarks.

2. VSI three-level control chart

Assume that a random sample of products is collected form a process and the quality of each product is quantified as a quality value according to the following three-level quality value function (QVF):

$$V = \begin{cases} \upsilon_1 & if\ the\ item\ is\ conforming, \\ \upsilon_2 & if\ the\ item\ is\ marginal, \\ \upsilon_3 & if\ the\ item\ is\ nonconforming, \end{cases} \tag{2.1}$$

where $0 \leq \upsilon_1 < \upsilon_2 < \upsilon_3$. Larger quality values imply lower quality of product. The state of the process can be described by the probability distribution of V denoted by $\mathbf{p} = [p_1\ p_2\ p_3]$, where $p_k = Pr(V = \upsilon_k)$, $k = 1, 2, 3$, and $p_1 + p_2 + p_3 = 1$. Thus the mean and standard deviation of V are $\mu_V = \sum_{k=1}^{3} \upsilon_k p_k$ and $\sigma_V = \sqrt{\sum_{k=1}^{3} \upsilon_k^2 p_k - \mu_V^2}$.

When the process is in-control, $\mathbf{p} = \mathbf{p}_0 = [p_{01}\ p_{02}\ p_{03}]$, where p_{01}, p_{02} and p_{03} are specified probabilities such that the quality of products can meet the desired level.

Let μ_0 and σ_0 be the mean and standard deviation of V, respectively, when the process is in-control. When a FRS Three-level Shewhart control charts is employed to monitor the process, a random sample of fixed size n_0, that is, $V_1, V_2, ..., V_{n_0}$ is taken from the process every h_0 hours, and the sample means $\bar{V} = \sum_{i=1}^{n_0} V_i/n_0$ are plotted in sequential order to form the three-level Control Chart and the chart signals as soon as $\bar{V}$ is upper than $UCL_0 = \mu_0 + k_0\sigma_0/\sqrt{n_0}$ or less than $LCL_0 = \max\{\mu_0 - k_0\sigma_0/\sqrt{n_0}, 0\}$, where k_0 is a positive constant. When the process is out-of-control, the probability distribution of V is assumed to be known and denoted by $\mathbf{p}_c$. In this status, the mean and standard deviation of V, represented by μ_c and σ_c, respectively.

The VSI three-level control chart is a modification of the FRS three-level Control Chart. Let h_1 and h_2 be two different sampling intervals, such that $0 < h_2 < h_1$ while keeping the sample size fixed at n. The value of sampling interval, for t^{th} rational subgroup, is given by:

$$h(t) = \begin{cases} h_1 & LWL \leq \bar{V}_{t-1} \leq UWL, \\ h_2 & 0therwise. \end{cases} \tag{2.2}$$

Let w be the width of the warning limit and k be the width of the control limit, both of them fixed and $0 < w < k$. The $\bar{V}$ values should be plotted in a chart with warning and control limits defined as follow:

$$\begin{aligned} LCL &= \max\{\mu_0 - k\sigma_0/\sqrt{n}, 0\}, \\ UCL &= \mu_0 + k\sigma_0/\sqrt{n}, \\ LWL &= \max\{\mu_0 - w\sigma_0/\sqrt{n}, 0\}, \\ UWL &= \mu_0 + w\sigma_0/\sqrt{n}. \end{aligned}$$

It is obvious that when $\mu_0 \leq w\sigma_0/\sqrt{n}$, then $\mu_0 \leq k\sigma_0/\sqrt{n}$ and so $LCL = LWL = 0$. The chart consists of three regions which are safe region, $I_1 = (LWL, UWL)$, warning region, $I_2 = (LCL, LWL) \cup (UWL, UCL)$, and action region, $I_3 = (-\infty, LCL) \cup (UCL, +\infty)$. A signal is produced when the sample point falls in the action region. If the sample point falls in the warning region, we get the process is in-control, but there is an evidence that a signal might occur. The sample point in the safe region means that the processes is in-control and there is no evidence that any signal might occur. In view of the above mentioned, In this paper, the design parameters of FRS three-level control chart are denoted by (n_0, h_0, k_0) and those of VSI three-level control chart by (n, h_1, h_2, w, k).

3. Statistical performance measure

The speed with which a control chart detects process mean and/or variance shifts measures its statistical efficiency. When the interval between samples is fixed, the speed can be measured by the average run length (ARL). If the interval between samples or size of samples varies from time to time, the performance can be measured by some criterion such as the average time to signal (ATS), the average number of false alarms (ANF), and the adjusted average time to signal ($AATS$). $AATS$ is defined to be the expected value of the time from process shifts to the time when chart signals. When a process is in-control, it is desirable that the mean time from the beginning of the process until a signal be long, which guarantees fewer false alarms. When a process is out-of-control, it is desirable that the mean time from the occurrence of the assignable cause until a signal be short as this guarantees the fast detection of process changes. Therefore, it is desirable to have small ANF and $AATS$.

If the assignable cause occurs according to an exponential distribution with parameter λ, $AATS$ can be obtained by

$$AATS = ATC - 1/\lambda, \tag{3.3}$$

where ATC is the average time of the cycle, i.e., the expected length of time from the start of process monitoring until the first signal after the process shift. Following Faraz and Saniga (2011), we use the Markov chain approach to calculate ANF and $AATS$. In t^{th} sampling stage, according to the status of the process (in or out of control) and the position of in the control chart (in the safe region, warning region, or action region), There are six different states as below.

1. $\bar{V}_t$ is in the safe region ($\bar{V}_t \in I_1$) and the process is in-control ($\mathbf{P}_t = \mathbf{P}_0$),
2. $\bar{V}_t$ is in the warning region ($\bar{V}_t \in I_2$) and the process is in-control ($\mathbf{P}_t = \mathbf{P}_0$),
3. $\bar{V}_t$ is in the action region ($\bar{V}_t \in I_3$) and the process is in-control ($\mathbf{P}_t = \mathbf{P}_0$),
4. $\bar{V}_t$ is in the safe region ($\bar{V}_t \in I_1$) and the process is out-of-control ($\mathbf{P}_t = \mathbf{P}_1$),
5. $\bar{V}_t$ is in the warning region ($\bar{V}_t \in I_2$) and the process is out-of-control ($\mathbf{P}_t = \mathbf{P}_1$),
6. $\bar{V}_t$ is in the action region ($\bar{V}_t \in I_3$) and the process is out-of-control ($\mathbf{P}_t = \mathbf{P}_1$),

where $\mathbf{P}_t$ is process status in t^{th} sampling stage. Therefore, a stochastic process $Y = \{Y(t)\}_{t=1,2,...}$ can be defined to describe these states as follows:

$$Y(t) = i \Leftrightarrow (\bar{V}_t \in I_i \ and \ \mathbf{P}_t = \mathbf{P}_0) \ \ , \ \ for \ i = 1, 2, 3$$
$$Y(t) = i \Leftrightarrow (\bar{V}_t \in I_{i-3} \ and \ \mathbf{P}_t = \mathbf{P}_1) \ \ , \ \ for \ i = 4, 5, 6$$

It is obvious that Y is a Markov chain with state space $E = \{1, 2, 3, 4, 5, 6\}$ and transition matrix $(r_{ij})_{i,j \in E}$ such that $r_{ij} = Pr(Y(t) = j|Y(t-1) = i)$, for $t = 1, 2, \ldots$. State 6 is the only absorbing state of Y.

We shall compute only r_{11}; The others can be computed similarly. It is easily to see that:

$$r_{11} = e^{-\lambda h_1} \times Pr(LWL < \bar{V}_t < UWL | \mathbf{p}_t = \mathbf{p}_0, h(t) = h_1). \tag{3.4}$$

A Central Limit Theorem approximation can be applied here. Since $V_1, V_2, ..., V_n$ are assumed to be $i.i.d.$ random variables, so when n is sufficiently large,

$$\begin{aligned} &Pr(LWL < \bar{V}_t < UWL | \mathbf{p}_t = \mathbf{p}_0, h(t) = h_1) \\ &\qquad \cong Pr(\frac{\sqrt{n}(LWL - \mu_0)}{\sigma_0} < Z < \frac{\sqrt{n}(UWL - \mu_0)}{\sigma_0}) \\ &\qquad = \begin{cases} 2\Phi(w) - 1 & if\ LWL > 0, \\ \Phi(w) & if\ LWL = 0, \end{cases} \end{aligned}$$

where Z has standard normal distribution with distribution function $\Phi(.)$. Therefore, transition probability matrix is given by

$$\mathbf{P} = \begin{bmatrix} q_1 a_1 & q_1(a_2 - a_1) & q_1(1 - a_2) & (1 - q_1)b_1 & (1 - q_1)(b_2 - b_1) & (1 - q_1)(1 - b_2) \\ q_2 a_1 & q_2(a_2 - a_1) & q_2(1 - a_2) & (1 - q_2)b_1 & (1 - q_2)(b_2 - b_1) & (1 - q_2)(1 - b_2) \\ q_2 a_1 & q_2(a_2 - a_1) & q_2(1 - a_2) & (1 - q_2)b_1 & (1 - q_2)(b_2 - b_1) & (1 - q_2)(1 - b_2) \\ 0 & 0 & 0 & b_1 & (b_2 - b_1) & (1 - b_2) \\ 0 & 0 & 0 & b_1 & (b_2 - b_1) & (1 - b_2) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where $q_i = \exp(-\lambda h_i)$, $i = 1, 2$, and

$$a_1 = \begin{cases} 2\Phi(w) - 1 & if\ LWL > 0, \\ \Phi(w) & if\ LWL = 0, \end{cases},$$

$$a_2 = \begin{cases} 2\Phi(k) - 1 & if\ LCL > 0, \\ \Phi(k) & if\ LCL = 0, \end{cases},$$

$$b_1 = \begin{cases} \Phi\big(\frac{\sqrt{n}d + w}{\delta}\big) - \Phi\big(\frac{\sqrt{n}d - w}{\delta}\big) & if\ LWL > 0, \\ \Phi\big(\frac{\sqrt{n}d + w}{\delta}\big) & if\ LWL = 0, \end{cases},$$

$$b_2 = \begin{cases} \Phi\big(\frac{\sqrt{n}d + k}{\delta}\big) - \Phi\big(\frac{\sqrt{n}d - k}{\delta}\big) & if\ LCL > 0, \\ \Phi\big(\frac{\sqrt{n}d + k}{\delta}\big) & if\ LCL = 0, \end{cases}$$

where $d = (\mu_0 - \mu_c)/\sigma_0$ and $\delta = \sigma_c/\sigma_0$. The average number of transitions to each transient state before true alarm signals is given by $\mathbf{b}^{'}(\mathbf{I} - \mathbf{Q})^{-1}$, where $\mathbf{Q}$ is the 5×5 matrix obtained from $\mathbf{P}$ on deleting the elements corresponding to the absorbing state, $\mathbf{I}$ is the identity matrix of order 5 and $\mathbf{b}^{'} = (p_1, p_2, p_3, 0, 0)$ is a vector of initial probabilities, with $p_1 + p_2 + p_3 = 1$. Thus $ATC = \mathbf{b}^{'}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{h}$ where $\mathbf{h}^{'} = (h_1, h_2, h_2, h_1, h_2)$ is the vector of sampling time intervals. Also, $ANF = \mathbf{b}^{'}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{f}$, such that $\mathbf{f}^{'} = (0, 0, 1, 0, 0)$. In this paper, following Faraz et al. (2010), the vector $\mathbf{b} \not\subset$ is set to $(0, 1, 0, 0, 0)$, for providing an extra protection and preventing problems that are encountered during start-up.

4. The cost model

To construct our process control model by a VSI 3-level control chart, we consider the following usual assumptions:

(1) The process quality is controlled by a VSI scheme for 3-level products.

(2) The process is characterized by an in-control state $\mathbf{P} = \mathbf{P}_0$.

(3) The state of process is shifted from $\mathbf{P} = \mathbf{P}_0$ to a known $\mathbf{P} = \mathbf{P}_c$ by only one single assignable cause.

(4) The assignable cause is assumed to occur according to a Poisson process with intensity of one occurrences per hour.

(5) The process is not self-correcting.

(6) The quality cycle starts with the in-control state and continues until the process is repaired after an out-of-control signal.

(7) During the search for an assignable cause, the process is shut down.

We use the cost model proposed by Costa and Rahim (2001), based on the Markov chain approach. Let T_0 and T_1 denote the time needed to investigate a false alarm and the time needed to search for and repair the assignable cause following a true alarm, respectively. Then the expected time of a quality cycle is

$$E(T) = ATC + T_0 \times ANF + T_1. \tag{4.5}$$

Now we assume V_0 and V_1 stand for the hourly profit earned when the process is operating in-control and out-of-control, respectively, C_0 is the average consequence cost of a false alarm, C_1 is the average cost to find the assignable cause and repair the process and s is the average cost for each inspected item. Then the expected net profit per quality cycle is determined as follows:

$$E(C) = \frac{V_0}{\lambda} + V_1 \times AATS - C_0 \times ANF - C_1 - s \times ANI, \tag{4.6}$$

The ANI is the expected number of inspected item until the chart signals which is given by $ANI = \mathbf{b}^{'}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{n}$, where $\mathbf{n}^{'} = (n, n, n, n, n)$.

According to this model, the expected loss per hour is determined as follows:

$$E(L) = V_0 - \frac{E(C)}{E(T)}. \tag{4.7}$$

The purpose of the design of the economic VSI three-level control chart is to find the value of control chart parameters, i.e. (n, h_1, h_2, w, k) so that $E(L)$ is minimized. We set $0.1 \leq h_2 < h_1 \leq 8$ to keep the chart practical. The sample size n is integer-valued and to guarantee a good normality approximation, it should be $n \geq n_{\min}$ and to keep the chart practical $n \leq n_{\max}$ with suitable choices for $n_{\min}$ and $n_{\max}$. We conducted a simulation study to choose an appropriate value for $n_{\min}$. We considered V with $v_1 = 0$, $v_2 = \nu$, $v_3 = 1$ and probability distribution $\mathbf{p} = [0.89, 0.08, 0.03]$. Thus we drawn 100 samples from this distribution with size $n = 20 : 10 : 200$. Then the statistics $\bar{Z} = \sqrt{n}(\bar{V} - \mu_V)/\sigma_V$ was computed for each sample. We test normality of these 100 values $\bar{Z}$ by Kolmogorov-Smirnov test at significant level 0.05. This operations repeated 10000 times for each of the values $\nu = 0.2$, $\nu = 0.5$ and $\nu = 0.99$. Finally, we computed the proportion of accepted normality hypothesis. The results are shown in Figure 1 and indicate that $n_{\min} = 80$ is a good choice.

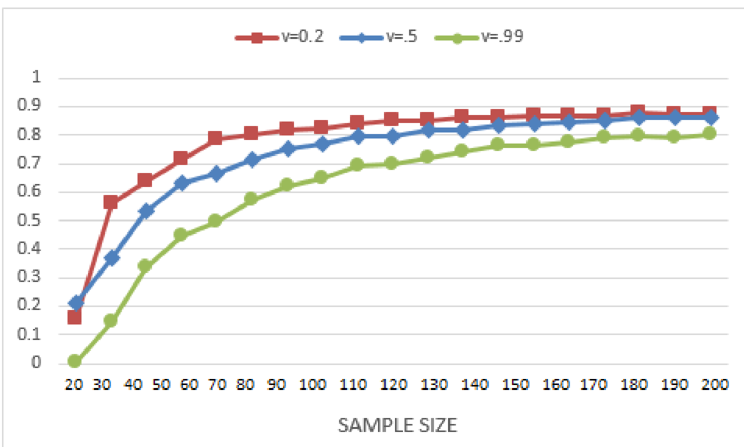

Figure 1: Proportion of accepted normality hypothesis for distribution of $\bar{V}$

In this paper, It is assumed that $n_{\min} = 80$ and $n_{\max} = 500$ to keep the chart practical and reliable. The other two parameters are real-valued such that $0 < w < k$. Therefore, a ED VSI three-level control chart can be modelled by the below optimization problem

$$\begin{aligned}
&\min E(L)\\
&s.t.\quad 0.1 \leq h_2 < h_1 \leq 8,\\
&0 \leq w \leq k,\\
&n \in\not\subset^+, n_{\min} \leq n \leq n_{\max}.
\end{aligned} \tag{4.8}$$

where $E(L)$ is given by (7). $AATS \leq AATS_0$ and $ANF \leq ANF_0$, where $AATS_0$ and ANF_0 are upper bound of adjusted average time to signal and average number of false alarms, respectively, are added to detect process shifts as quickly as possible and to form the best protection against false alarms. By adding these constraints to the optimization problem (8), an $ESD\ VSI$ three-level control chart are achieved.

The solution procedure is carried out using the genetic algorithm to obtain the optimal values of n, h_1, h_2, w , and k. The $MATLAB$ software (version R2013a) is used to run the GA.

5. Numerical Comparisons

The purpose of this section is to design a VSI three-level control chart and compare it to the FRS three-level control chart with respect to economic and statistical criteria. For this purpose, We consider a QVF with $\upsilon_1 = 0$, $\upsilon_2 = \nu$, and $\upsilon_3 = 1$. Let probability vector of in-control state be $\mathbf{p}_0 = (0.89, 0.08, 0.03)$. The corresponding mean and standard deviation are presented in Table 1. Three probability vectors of out-of-control state are considered as given in Table 1, to simulate three types of scenario of process shifts: very small ($d < 0.1$), small (0.1 £ $d < 0.19$) and moderate (0.19 £ d £ 1) shifts, which are denoted by A, B and C, respectively. Pourtaheri (2017) noticed that a relevant choice is $\upsilon = 0.99$ for very small shifts, and $\upsilon = 0.2$ for small and moderate shifts.

Table 2 gives the ten process and cost parameters which are reported by Costa and Rahim (2001). These values provide a general variation in parameter values and have been used here to compute the loss function given in (7).

The ESD three-level control charts (VSI and FRS) were obtained by solving the optimization problem (8) to which added the statistical constraints $AATS \leq 7$ and $ANF \leq 0.5$. Table 3 shows the optimal parameters of design and the corresponding costs for FRS and VSI schemes. The results indicate that VSI chart is more economical than FRS, especially for very small and small shifts. In

Table 1: Values of μ_V and σ_V in various status of process for different values of υ

p					υ							
					0.2				0.99			
		p_1	p_2	p_3	μ_V	σ_V	d	δ	μ_V	σ_V	d	δ
p_0		0.89	0.08	0.03	0.046	0.176	0	1	0.109	0.31	0	1
	A	0.87	0.1	0.03	0.05	0.178	0.02	1.007	0.13	0.33	0.06	1.07
p_c	B	0.85	0.1	0.05	0.07	0.222	0.14	1.257	0.15	0.35	0.13	1.14
	C	0.83	0.1	0.07	0.09	0.257	0.25	1.456	0.17	0.37	0.19	1.20

Table 2: The 10 processes and cost parameters

No.	s	C_0	C_1	V_0	V_1	T_0	T_1	λ
1	5	500	500	500	50	5.0	1	0.01
2	10	500	500	500	50	5.0	1	0.01
3	5	250	500	500	50	5.0	1	0.01
4	5	500	50	500	50	5.0	1	0.01
5	5	500	500	250	50	5.0	1	0.01
6	5	500	500	500	100	5.0	1	0.01
7	5	500	500	500	0	5.0	1	0.01
8	5	500	500	500	50	2.5	1	0.01
9	5	500	500	500	50	5.0	10	0.01
10	5	500	500	500	50	5.0	1	0.05

fact, VSI scheme leads to about 54%, 19% and 5% cost improvements per hour for shifts in level A, B and C, respectively.

6. Conclusions

In this paper, an economic-statistical design of three-level control chart with variable sampling intervals have been presented based on the Markov chain approach. The cost model is due to Costa and Rahim (2001) and genetic algorithm were applied to find the optimal five chart parameters $(n, h_1, h_2, w_\nu, k_\nu)$. Extensive numerical comparison has been done between $ESD\ VSI$ and $ESD\ FRS$ three-level control chart. The results indicate that VSI scheme is more economic than FRS one, in very small, small and moderate shifts.

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Table 3: The optimal parameters of the $ESD\ VSI$ three-level scheme.

$SH.$	$NO.$	FRS					VSS						
		n	h	k	$AATS$	$E(L)$	n	h_1	h_2	w	k	$AATS$	$E(L)$
	1	493	0.48	3.03	7	5059.66	491	2.08	0.10	0.89	2.72	7	1925.04
	2	491	0.48	3.03	7	9889.68	492	3.68	0.10	0.44	2.72	7	3925.17
	3	490	0.47	3.03	7	5121.38	490	2.10	0.10	0.87	2.73	7	1924.06
	4	491	0.47	3.04	7	5097.23	491	3.22	0.10	0.53	2.71	7	1906.53
	5	490	0.47	3.04	7	5100.63	499	2.65	0.10	0.72	2.71	7	1793.76
A	6	491	0.47	3.03	7	5097.76	491	2.50	0.11	0.75	2.70	7	1867.48
	7	490	0.47	3.04	7	5125.08	491	2.19	0.10	0.85	2.72	7	1906.79
	8	497	0.49	3.02	7	5030.08	492	5.53	0.14	0.20	2.84	7	2943.07
	9	490	0.47	3.04	7	4776.98	492	2.42	0.10	0.76	2.70	7	1770.63
	10	490	3.66	1.64	7	778.78	491	4.64	2.48	1.20	1.29	7	813.93
	1	83	1.17	2.52	7	393.72	297	7.89	0.81	1.30	2.15	7	288.66
	2	89	1.30	2.49	7	716.93	82	3.17	0.11	0.76	2.37	7	511.43
	3	80	1.11	2.54	7	399.65	146	3.94	0.19	1.19	2.47	7	305.86
	4	89	1.31	2.48	7	376.75	125	3.36	0.11	1.02	2.52	7	316.70
	5	85	1.22	2.50	7	365.49	93	2.73	0.11	0.89	2.53	7	294.17
B	6	83	1.17	2.52	7	390.54	100	2.77	0.11	0.86	2.61	7	338.27
	7	82	1.15	2.52	7	399.38	90	2.37	0.12	1.08	2.48	7	312.40
	8	80	1.11	2.54	7	399.59	296	7.41	0.86	1.44	2.18	7	289.07
	9	81	1.13	2.53	7	406.00	117	3.48	0.11	0.85	2.56	7	340.45
	10	82	2.91	1.76	7	309.98	89	4.11	2.96	0.34	1.60	7	307.83
	1	86	4.93	2.24	7	135.59	100	7.64	1.87	1.36	2.15	7	127.97
	2	80	4.54	2.27	7	222.06	108	7.58	3.39	1.44	2.11	7	207.66
	3	82	4.69	2.26	7	135.13	115	8.00	3.41	1.30	2.19	7	129.43
	4	83	4.73	2.25	7	131.93	81	6.26	0.20	1.53	2.37	7	118.49
	5	83	4.73	2.26	7	112.24	112	7.98	4.70	1.11	2.10	7	105.55
C	6	80	4.32	2.33	7	131.79	112	8.00	3.34	1.37	2.11	7	126.30
	7	82	4.67	2.26	7	139.31	107	7.72	3.69	1.17	2.12	7	131.83
	8	83	4.73	2.26	7	131.85	110	7.65	4.16	1.33	2.11	7	125.19
	9	91	5.25	2.21	7	162.52	87	6.58	0.63	1.34	2.45	7	153.93
	10	80	7.33	1.48	7	221.91	80	7.98	6.95	0.22	1.55	7	222.59

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