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A Mahalanobis-Distance-Based Copula Model Averaging Approach for Dependence Modelling with an Application to the Tehran Province Earthquake Catalogue

Sedigheh Shams^{*1}, Mahnaz Rahmani²

¹Department of Statistics, Faculty of Mathematical Sciences, Alzahra University

²Department of Statistics, Faculty of Mathematical Sciences, Alzahra University

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Abstract: Copula models provide a flexible framework for modelling complex dependence structures between continuous variables. However, several competing copula families may provide similarly satisfactory fits, making inference based on a single selected model potentially unstable. To address this issue, a Mahalanobis-distance-based copula model-averaging approach is adopted to account for model uncertainty and obtain more stable estimates of conditional exceedance probabilities. Its performance is evaluated through simulation studies across a range of Kendall's τ values and illustrated using an earthquake catalogue from Tehran Province, Iran, to examine dependence between earthquake magnitude and inter-event time. The simulation results demonstrate stable estimation across different dependence structures. In the real-data application, appropriate marginal models are fitted before applying the copula framework, and the estimated conditional exceedance probabilities vary only slightly across elapsed-time thresholds. These findings suggest that elapsed time alone provides limited predictive information about the magnitude of subsequent earthquakes and highlight the value of incorporating model uncertainty into copula-based dependence modelling.

Keywords: Copula model; Mahalanobis distance; Model averaging; Dependence modelling; Earthquake catalogue.

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^{*}Corresponding Author: s.shams@alzahra.ac.ir

1. Introduction

Modelling dependence between random variables is a fundamental problem in statistics and data science, with important applications in finance, insurance, environmental sciences, reliability engineering, hydrology, and geosciences. Reliable dependence modelling is essential for understanding complex multivariate phenomena, quantifying joint risks, and making probabilistic predictions. Classical dependence measures, such as Pearson's correlation coefficient, are often inadequate for describing nonlinear or asymmetric relationships that frequently arise in practice. Copula models overcome these limitations by separating the marginal distributions from their dependence structure, thereby providing a flexible and theoretically sound framework for modelling complex multivariate dependence.

A practical challenge in copula modelling is the presence of model uncertainty. Numerous copula families have been proposed, and different copulas often provide similarly satisfactory fits to the same dataset. Consequently, selecting a single copula model may lead to unstable statistical inference and conclusions that depend heavily on the adopted model selection criterion. Model averaging provides an attractive alternative by combining information from multiple candidate models and explicitly accounting for model uncertainty, thereby improving the robustness of dependence estimation.

An important application of dependence modelling arises in statistical seismology, where a long-standing question is whether the time elapsed since a previous earthquake contains useful information about the magnitude of the subsequent event. This question forms the basis of recurrence-based concepts such as the seismic gap hypothesis and has important implications for seismic hazard assessment, earthquake forecasting, and long-term risk evaluation. Despite extensive research, empirical evidence remains inconclusive, and the reported relationship between earthquake magnitude and elapsed time varies considerably across tectonic environments and statistical modelling approaches.

Various statistical models have been proposed to describe earthquake recurrence. Mixed distributions have been used to separate clustered seismicity from background occurrence (Talbi and Yamazaki, 2010), while time-dependent stochastic recurrence models have been developed to estimate the occurrence probabilities of large earthquakes on individual fault systems (Kourouklas *et al.*, 2018). Kagan and Jackson (1995) developed magnitude forecasts based on a modified seismic gap hypothesis, whereas Dargahi-Noubary and Razzaghi (1994) proposed a bivariate exponential model for the joint distribution of earthquake magnitude and elapsed time. More recently, Husker *et al.* (2023) reported encouraging predictive performance of seismic-gap-based recurrence models in selected regions, whereas Lippiello *et al.* (2009) demonstrated that earthquake occurrence exhibits complex

clustering behaviour in both space and time. Temporal changes in earthquake magnitude distributions during foreshock sequences were investigated by [Apostol and Cune \(2023\)](#), while [Petrillo and Zhuang \(2023\)](#) examined statistical evidence for magnitude dependence in earthquake occurrence. These studies collectively suggest that the existence and strength of dependence between earthquake magnitude and elapsed time remain open questions.

Copula models provide a natural framework for investigating such dependence because they allow the marginal distributions and the dependence structure to be modelled separately. Recent developments continue to extend copula methodology in both theory and applications ([Genest *et al.* \(2024\)](#)).

Recent research continues to extend copula methodology to increasingly complex dependence structures, including dynamic and state-space models ([Hanebeck *et al.* \(2025\)](#)).

However, relatively few studies have employed copula-based methods for earthquake recurrence analysis, and most existing analyses rely on a single selected copula model.

To account for model uncertainty, the present study adopts the model averaging strategy introduced by [Nikoloulopoulos and Karlis \(2008\)](#) to investigate the dependence between earthquake magnitude and elapsed time in the earthquake catalogue of Tehran Province, Iran. In addition to the real-data application, comprehensive simulation studies are conducted over a range of Kendall's τ values to evaluate the performance of the adopted framework under different levels of dependence.

Tehran Province is located within the Alpine–Himalayan seismic belt and is influenced by several major active fault systems, including the North Tehran, Mosha, and Rey faults. Owing to its high seismic activity and tectonic complexity, it provides an appropriate case study for examining the relationship between earthquake magnitude and elapsed time using flexible dependence models.

The objectives of this study are threefold. First, we investigate the performance of a Mahalanobis-distance-based copula model averaging approach for dependence modelling through simulation studies over different values of Kendall's τ . Second, we apply the adopted framework to the earthquake catalogue of Tehran Province to obtain model-averaged estimates of dependence measures and conditional exceedance probabilities while accounting for model uncertainty. Third, we provide empirical evidence regarding the relationship between earthquake magnitude and elapsed time and discuss its implications for the seismic gap hypothesis.

The principal methodological contribution of this study is the extension of the Mahalanobis-distance-based copula model averaging framework of [Nikoloulopoulos and Karlis \(2008\)](#). Specifically, we provide an asymptotic justification for the

Mahalanobis-distance selection criterion, establish the stability of the resulting selection-frequency weights, and examine through simulation and earthquake data how explicitly accounting for model uncertainty affects the estimation of conditional exceedance probabilities relative to reliance on a single copula model.

The remainder of this paper is organised as follows. Section 2 introduces the copula methodology and the Mahalanobis-distance-based model averaging procedure. Section 3 presents the simulation study. Section 4 describes the earthquake catalogue and reports the real-data analysis. Section 5 discusses the results and their implications.

2. Methodology

This section outlines the copula function, dependence structure, the model averaging technique, and the threshold selection method.

2.1 Copula Function

A bivariate copula is a distribution function on $[0, 1]^2$ with uniform margins. By Sklar's theorem (Nelsen, 2006), for continuous marginal distribution functions F and G , the joint distribution of (X, Y) can be represented uniquely as

$$H(x, y) = C\{F(x), G(y)\}, \quad (2.1)$$

where the copula C describes the dependence structure independently of the marginal distributions.

The copula families considered in this study belong mainly to the Archimedean and elliptical classes. Archimedean copulas provide flexible forms of asymmetric dependence, whereas elliptical copulas are useful for representing symmetric dependence structures. The particular families relevant to the analysis and their tail-dependence properties are summarized in Table 1; further theoretical details can be found in Nelsen (2006).

When the copula density $c(u, v)$ exists, the joint density can be written as

$$h_{X,Y}(x, y) = c\{F(x), G(y)\} f(x) g(y), \quad (2.2)$$

where f and g are the marginal densities. Conditional on the fitted margins, the copula parameter θ can then be estimated by maximizing the copula log-likelihood

$$\ell(\theta) = \sum_{i=1}^n \log c(u_i, v_i; \theta), \quad u_i = F(x_i), \quad v_i = G(y_i). \quad (2.3)$$

This two-step formulation separates marginal modelling from dependence estimation and is commonly used in parametric copula analysis (Joe, 2005).

2.2 Dependence Structure

The choice of a copula family should reflect the main dependence features of the data. In the present study, Kendall's τ is used to describe overall monotone dependence, while the upper and lower tail-dependence coefficients are used to assess association in the extremes. These measures are particularly relevant here because the candidate copulas differ substantially in their tail behaviour (Nelsen, 2006).

For a copula C , Kendall's τ can be expressed as

$$\tau_C = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1. \quad (2.4)$$

Positive and negative values indicate concordant and discordant dependence, respectively, while values near zero indicate weak monotone association. The upper and lower tail-dependence coefficients are defined, respectively, by

$$\lambda_U = \lim_{u \rightarrow 1^-} P(V > u \mid U > u) = \lim_{u \rightarrow 1^-} \frac{1 - 2u + C(u, u)}{1 - u}, \quad (2.5)$$

and

$$\lambda_L = \lim_{u \rightarrow 0^+} P(V \leq u \mid U \leq u) = \lim_{u \rightarrow 0^+} \frac{C(u, u)}{u}. \quad (2.6)$$

These coefficients distinguish copulas with upper-tail, lower-tail, symmetric-tail, or no asymptotic tail dependence and therefore provide a direct basis for comparing candidate dependence structures in the earthquake application.

Table 1 summarizes the candidate copula families and their main characteristics.

Table 1: Candidate copula families and their main characteristics.

Copula	$C_\theta(u, v)$	Parameter space	Tail dependence
Gaussian	$\Phi_\Sigma(\Phi^{-1}(u), \Phi^{-1}(v))$	$\rho \in (-1, 1)$	$\lambda_L = \lambda_U = 0$
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$	$\theta \in [-1, \infty) \setminus \{0\}$	$\lambda_L = 2^{-1/\theta}, \lambda_U = 0$
Frank	$-\frac{1}{\theta} \log \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right)$	$\theta \neq 0$	$\lambda_L = \lambda_U = 0$
Gumbel	$\exp \left[- \left((-\log u)^\theta + (-\log v)^\theta \right)^{1/\theta} \right]$	$\theta \geq 1$	$\lambda_L = 0, \lambda_U = 2 - 2^{1/\theta}$
Joe	$1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]^{1/\theta}$	$\theta \geq 1$	$\lambda_L = 0, \lambda_U = 2 - 2^{1/\theta}$
AMH	$\frac{uv}{1 - \theta(1-u)(1-v)}$	$-1 \leq \theta < 1$	$\lambda_L = \lambda_U = 0$

2.3 Model Averaging Technique

The principal methodological focus of this study is to account for copula-model uncertainty when estimating conditional exceedance probabilities. Rather than selecting a single copula and conditioning all subsequent inference on that choice, we extend the model-selection procedure of [Nikoloulopoulos and Karlis \(2008\)](#) by combining Mahalanobis-distance-based selection with model averaging.

The procedure evaluates the stability of competing copula models over non-parametric bootstrap replications. In each replication, the model with the smallest Mahalanobis distance is selected, and the resulting selection frequencies are used as model-averaging weights. Thus, the final conditional exceedance estimate incorporates uncertainty arising from the choice of the dependence model rather than relying exclusively on a single selected copula. This selection-frequency formulation is related to the general model-uncertainty framework of [Buckland *et al.* \(1997\)](#).

Suppose that m candidate copula models are under consideration. For the j th copula, let $\hat{\theta}_j$ denote the maximum likelihood estimate obtained from the observed data. A nonparametric bootstrap with B resamples is then performed, producing the bootstrap estimates

$$\hat{\theta}_{j1}, \dots, \hat{\theta}_{jB}.$$

These bootstrap replicates are used to estimate the centre and dispersion of the sampling distribution through $\bar{\theta}_j = \frac{1}{B} \sum_{l=1}^B \hat{\theta}_{jl}$, and $S_j = \frac{1}{B-1} \sum_{l=1}^B (\hat{\theta}_{jl} - \bar{\theta}_j)(\hat{\theta}_{jl} - \bar{\theta}_j)'$.

The agreement between the original estimate and the corresponding bootstrap distribution is measured by the squared Mahalanobis distance

$$D_j^2 = (\hat{\theta}_j - \bar{\theta}_j)' S_j^{-1} (\hat{\theta}_j - \bar{\theta}_j). \quad (2.7)$$

A small value of D_j^2 indicates that the estimate obtained from the observed sample is well supported by its bootstrap distribution, whereas larger values suggest poorer agreement. Consequently, the Mahalanobis distances provide a natural basis for assigning weights to competing copula models in the model averaging procedure described below.

The following regularity conditions are assumed in establishing the asymptotic properties of the proposed selection-frequency weights and the Mahalanobis-distance diagnostic:

(A1) For every candidate copula model, the bootstrap maximum likelihood estimator is consistent for the corresponding true parameter vector; that is, $\hat{\theta}_{jl} \xrightarrow{p} \theta_j^*$, as $n \rightarrow \infty$.

(A2) The bootstrap covariance matrix associated with each candidate model is asymptotically nonsingular.

(A3) The standard regularity conditions ensuring the asymptotic normality of the maximum likelihood estimator and the validity of the nonparametric bootstrap are satisfied.

Proposition 1. (Stability of Mahalanobis-distance selection.)

Under (A1)–(A3), for each fixed model j we have, as $n, B \rightarrow \infty$,

$$D_j^2 \xrightarrow{p} 0 \quad \text{whenever } \hat{\theta}_j - \bar{\theta}_j = o_p(1),$$

and D_j^2 remains bounded away from zero if $\|\hat{\theta}_j - \bar{\theta}_j\|$ does not vanish while S_j remains well-conditioned; it may diverge if the discrepancy grows.

Moreover, the selection-frequency weights

$$w_j = \frac{1}{T} \# \{\text{replications for which model } j \text{ attains the smallest } D_j^2\}$$

converge (in probability) to limiting frequencies that place higher mass on the most stable models.

Proof.

By (A1), both $\hat{\theta}_j$ and $\bar{\theta}_j$ converge to θ_j^* , and hence $\hat{\theta}_j - \bar{\theta}_j = o_p(1)$ for stable models. Under (A2)–(A3), $S_j \xrightarrow{p} \Sigma_j$ for some positive definite matrix Σ_j , so $S_j^{-1} = O_p(1)$. Therefore,

$$D_j^2 = (\hat{\theta}_j - \bar{\theta}_j)' S_j^{-1} (\hat{\theta}_j - \bar{\theta}_j) = o_p(1).$$

If $\|\hat{\theta}_j - \bar{\theta}_j\|$ does not vanish while $S_j^{-1} = O_p(1)$, then D_j^2 is bounded away from zero (and may diverge if the discrepancy grows). The final statement follows because, across replications, models with smaller D_j^2 are selected more frequently, and the empirical selection frequencies converge to their probabilities by the law of large numbers. $\square$

Proposition 1 provides a theoretical foundation for using selection frequencies as model-averaging weights. It does not establish the optimality of the weighting scheme, but rather demonstrates its stability under standard regularity conditions. This result provides theoretical support for the proposed weighting strategy.

2.4 Threshold Selection Method

To model the tail behaviour of earthquake magnitudes, the peaks-over-threshold (POT) approach is adopted (Coles, 2001). Let X denote the earthquake magnitude and let u be a sufficiently high threshold. The exceedances above the threshold are defined by

$$Y = X - u, \quad X > u.$$

According to the Pickands–Balkema–de Haan theorem (Coles, 2001), for a sufficiently large threshold u , the conditional distribution of the exceedances can be approximated by a generalized Pareto distribution (GPD),

$$G(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi}, & \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\sigma}\right), & \xi = 0, \end{cases} \quad (2.8)$$

where $\sigma > 0$ and ξ denote the scale and shape parameters, respectively.

A practical issue in POT modelling is the selection of an appropriate threshold. Choosing a threshold that is too low may violate the asymptotic assumptions underlying the GPD approximation and introduce bias, whereas an excessively high threshold leaves too few exceedances for reliable estimation, resulting in increased variance. Consequently, threshold selection represents one of the most important practical issues in extreme value analysis (Benito *et al.*, 2023).

A key theoretical property of the generalized Pareto distribution is threshold stability. If the exceedances above a threshold u_0 follow a GPD, then the exceedances above any higher threshold $u \geq u_0$ also follow a GPD with the same shape parameter ξ , while the corresponding scale parameter satisfies $\sigma_u = \sigma_{u_0} + \xi(u - u_0)$. Consequently, the theoretical mean excess function is

$$e(u) = E(X - u \mid X > u) = \frac{\sigma_u}{1 - \xi}, \quad \xi < 1, \quad (2.9)$$

which is linear in the threshold u . This property forms the theoretical basis of the mean residual life (MRL) plot for threshold selection.

For a given threshold u , the empirical mean excess function is estimated by $\hat{e}(u) = \frac{1}{n_u} \sum_{X_i > u} (X_i - u)$, where n_u denotes the number of observations exceeding u .

The empirical MRL plot is constructed by plotting $\hat{e}(u)$ against a sequence of candidate thresholds. The selected threshold is taken as the smallest value beyond which the plot exhibits an approximately linear trend, indicating that the GPD provides an adequate approximation to the tail distribution.

After selecting the threshold, the adequacy of the fitted GPD is assessed using probability (PP) and quantile (QQ) plots. A satisfactory fit is indicated when the plotted points lie close to the reference line, suggesting close agreement between the empirical exceedances and the fitted GPD. The fitted GPD is then used as the marginal distribution of the earthquake magnitude variable.

3. Simulation Study

The purpose of this simulation study is to assess how closely the proposed Mahalanobis-distance-based model-averaging procedure can estimate a conditional exceedance probability when the underlying dependence structure is known. Four copula families, Clayton, Frank, Gumbel, and Student's t copula, were considered as data-generating models. These families represent different dependence and tail structures: the Clayton copula exhibits lower-tail dependence, the Gumbel copula exhibits upper-tail dependence, the Frank copula has no tail dependence, and the t copula allows symmetric tail dependence.

The strength of dependence was specified through Kendall's τ . For positive dependence, $\tau \in \{0.1, 0.5, 0.9\}$ was considered for all four copula families. For negative dependence, $\tau \in \{-0.1, -0.5, -0.9\}$ was considered only for the Frank and t copulas, since the standard Clayton and Gumbel copulas used here do not accommodate the corresponding negative dependence.

For each combination of the data-generating copula and Kendall's τ , a single sample of size $n = 100$ was generated. Several copula families were fitted to the generated sample, and their adequacy was assessed using the Anderson–Darling goodness-of-fit test. The copula families that were not rejected by the goodness-of-fit test were retained as the candidate copulas, denoted by $C_1, \dots, C_m$.

The Mahalanobis-distance-based model-averaging procedure described in Section 2.3 was then applied to the retained candidate copulas. For each candidate copula C_j , the parameter vector $\hat{\theta}_j$ was estimated from the original sample. Within each replication, a nonparametric bootstrap with $B = 200$ resamples was performed. The candidate copulas were fitted to each bootstrap resample, yielding the bootstrap parameter estimates $\hat{\theta}_{j1}, \dots, \hat{\theta}_{jB}$. These estimates were used to calculate the bootstrap mean $\bar{\theta}_j$ and covariance matrix S_j , and hence the squared Mahalanobis distance D_j^2 for each candidate copula.

The selection procedure was repeated $T = 1000$ times. In each replication, the candidate copula attaining the smallest squared Mahalanobis distance was selected. As defined in Proposition 1, the model-averaging weight of candidate copula C_j was calculated as its selection frequency,

$$w_j = \frac{1}{T} \# \{ \text{replications for which model } j \text{ attains the smallest } D_j^2 \}.$$

The resulting selection-frequency weights were then used to construct the model-averaged copula.

The quantity of interest was the conditional exceedance probability

$$p_{\text{true}} = P(U > u^* \mid V > v^*) = \frac{1 - u^* - v^* + C_0(u^*, v^*)}{1 - v^*}, \quad (3.10)$$

where C_0 denotes the data-generating copula. The probability was evaluated at $u^* = v^* = 0.8$. This choice places the evaluation point in the upper part of the unit square while avoiding an excessively extreme threshold for samples of size $n = 100$.

For each replication, the conditional exceedance probability was estimated using the proposed model-averaging method (A). If $\hat{C}_{jr}$ denotes the fitted j th candidate copula in replication r and w_{jr} its corresponding model-averaging weight, the averaged copula can be written as

$$\hat{C}_{A,r}(u, v) = \sum_{j=1}^m w_{jr} \hat{C}_{jr}(u, v), \quad \sum_{j=1}^m w_{jr} = 1. \quad (3.11)$$

The corresponding estimate of the conditional exceedance probability is

$$\hat{p}_{A,r} = \frac{1 - u^* - v^* + \hat{C}_{A,r}(u^*, v^*)}{1 - v^*}. \quad (3.12)$$

For comparison, a single-model estimator (S) was also evaluated over the same replications. Its role is to provide a reference benchmark under the same repeated-sampling scheme as the model-averaging estimator, rather than to define the primary target of the simulation. The main objective is to compare the model-averaged estimate with the known value p_{true} .

For each method, the mean, variance, and bias were calculated over the $T = 1000$ replications. In particular, for the model-averaging estimator,

$$\bar{p}_A = \frac{1}{T} \sum_{r=1}^T \hat{p}_{A,r}, \quad \text{Bias}(A) = \bar{p}_A - p_{\text{true}}. \quad (3.13)$$

Thus, the closeness of $\bar{p}_A$ to p_{true} measures the accuracy of the proposed procedure, whereas the empirical variance describes its stability across repeated samples.

Simulation Results

Table 2 summarizes the simulation performance of the single-model reference estimator (S) and the proposed model-averaging estimator (A) for the different data-generating copulas and dependence levels.

The primary purpose of Table 2 is to examine how closely the model-averaging estimator reproduces the known conditional exceedance probability p_{true} . The results show that its accuracy depends on both the data-generating copula and the strength of dependence.

For the Frank copula, the model-averaged estimates are close to the true conditional probabilities over the considered positive dependence levels. For example, when $\tau = 0.5$, $p_{\text{true}} = 0.5490$ and the mean of the model-averaging estimator

Table 2: Simulation performance of single-model (S) and model-averaging (A) estimators of the conditional exceedance probability under different true copula families and dependence levels measured by Kendall's τ_{true} .

Family	τ_{true}	p_{true}	Mean (S)	Var (S)	Bias (S)	Mean (A)	Var (A)	Bias (A)
Frank	0.1	0.2605	0.2747	0.0010	0.0142	0.2742	0.0006	0.0137
Gumbel	0.1	0.2971	0.2746	0.0009	-0.0225	0.2745	0.0006	-0.0225
Clayton	0.1	0.2339	0.2765	0.0010	0.0426	0.2764	0.0007	0.0425
t	0.1	0.2971	0.2769	0.0010	-0.0202	0.2761	0.0007	-0.0210
Frank	0.5	0.5490	0.5621	0.0051	0.0131	0.5608	0.0025	0.0118
Gumbel	0.5	0.6469	0.5651	0.0047	-0.0818	0.5621	0.0025	-0.0847
Clayton	0.5	0.4300	0.5661	0.0046	0.1362	0.5628	0.0024	0.1328
t	0.5	0.5891	0.5658	0.0044	-0.0233	0.5626	0.0022	-0.0265
Frank	0.9	0.9095	0.9070	0.0008	-0.0025	0.9063	0.0004	-0.0032
Gumbel	0.9	0.9364	0.9072	0.0008	-0.0292	0.9063	0.0003	-0.0302
Clayton	0.9	0.8508	0.9084	0.0007	0.0575	0.9070	0.0003	0.0562
t	0.9	0.9163	0.9088	0.0006	-0.0074	0.9070	0.0003	-0.0093
Frank	-0.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
t	-0.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Frank	-0.5	0.0129	0.0216	0.0001	0.0087	0.0212	0.0000	0.0082
t	-0.5	0.0267	0.0220	0.0001	-0.0047	0.0216	0.0000	-0.0051
Frank	-0.1	0.1457	0.1624	0.0005	0.0167	0.1623	0.0004	0.0166
t	-0.1	0.1773	0.1617	0.0005	-0.0156	0.1619	0.0004	-0.0154

is 0.5608, corresponding to a bias of 0.0118. Under strong positive dependence ($\tau = 0.9$), the biases for the Frank and t copulas are -0.0032 and -0.0093 , respectively, indicating close agreement with the corresponding true probabilities.

The results also identify settings in which the approximation is less accurate. In particular, at the moderate dependence level $\tau = 0.5$, the biases of the model-averaging estimator are -0.0847 for the Gumbel copula and 0.1328 for the Clayton copula. Therefore, the simulation does not support a claim of uniformly negligible bias across all dependence structures. Rather, the accuracy of the model-averaging estimator depends on the form and strength of the underlying dependence, with larger discrepancies occurring in some asymmetric dependence settings.

For negative dependence, considered for the Frank and t copulas, the model-averaging estimates remain reasonably close to the corresponding true conditional probabilities. When $\tau = -0.9$, both the true and estimated conditional exceedance probabilities are zero at the selected threshold $u^* = v^* = 0.8$.

Table 2 also shows that the variance of the model-averaging estimator is smaller than the empirical variance of the single-model reference estimator in the reported scenarios. For example, for the Frank copula with $\tau = 0.5$, the variance decreases from 0.0051 for S to 0.0025 for A. Similar variance reductions occur across the other reported settings. This indicates greater repeated-sampling stability of the

averaged estimator. However, the Clayton and Gumbel results at $\tau = 0.5$ also demonstrate that a reduction in variance does not necessarily imply a reduction in bias.

Overall, the simulation results show that the proposed model-averaging procedure can provide a stable approximation to the true conditional exceedance probability, but its accuracy is not uniform across all dependence structures. The results therefore support a more qualified interpretation of the method: model averaging reduces sampling variability in the reported scenarios, while the remaining bias can depend on the underlying copula family and the strength of dependence.

4. Study Region and Earthquake Catalogue

Iran is located within the Alpine–Himalayan seismic belt and is characterized by complex tectonic interactions between the Arabian and Eurasian plates. Major fault systems such as the Zagros and Alborz faults contribute to significant seismic activity across the region.

Tehran Province lies within the southern Alborz range, a complex transpressional system shaped by the convergence of the Arabian and Eurasian plates.

Major active structures, including the North Tehran Fault (NTF), Moshafasham Fault (MFF), and the Ray–Kahrizak fault system, exhibit distributed deformation and variable slip rates (Allen *et al.* (2003)).

Figure 1 shows the local seismicity and active faults in the Tehran region.

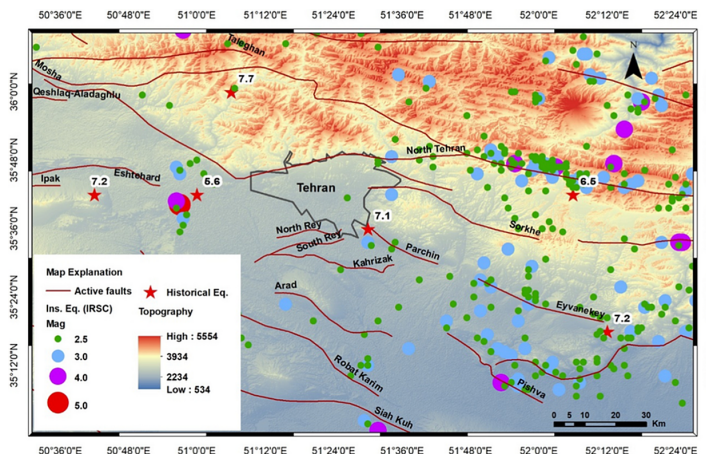


Figure 1: Active faults and historical (red stars) and instrumental (coloured circles) seismicity in the Tehran region. Adapted from Alikhanzadeh and Zafarani (2023).

The tectonic complexity in this region provides a geological explanation for why magnitude–elapsed time relationships may be inherently weak, supporting the statistical findings of this study.

Careful regional catalogue construction and preliminary seismicity characterization are essential before applying statistical models to earthquake occurrence data (Hinzen *et al.*, 2021).

The earthquake catalogue analysed in this study contains 634 events recorded between 19 August 1993 and 6 January 2015 within the geographic extent 34.88–36.57°N and 50.36–52.50°E. Each record includes the occurrence time, epicentral coordinates, focal depth, and local magnitude. The catalogue forms the basis for all subsequent dependence analyses.

Figure 2 presents the Gutenberg–Richter frequency–magnitude relationship for the study catalogue. The approximately linear trend provides a descriptive characterization of the earthquake magnitudes and serves as a preliminary assessment of the catalogue before the marginal and dependence analyses (Perez-Oregon *et al.*, 2018).

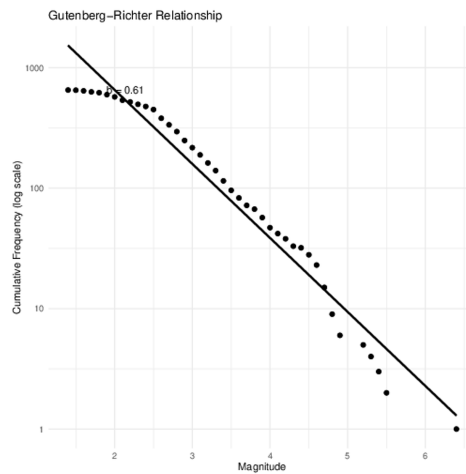


Figure 2: Gutenberg–Richter frequency–magnitude relationship for the Tehran Province earthquake catalogue.

Following the preliminary description of the study region and earthquake catalogue, the proposed copula-based methodology was applied to earthquakes exceeding the selected magnitude threshold.

4.1 Statistical Modelling

Following the preliminary examination of the earthquake catalogue, the marginal distributions and dependence structure were modelled using the proposed copula framework. Before copula estimation, possible large-scale spatial trends in earthquake magnitude were examined.

4.1.1 Threshold Justification

The threshold for the peaks-over-threshold analysis was selected following the procedure described in Section 2.4. Specifically, the mean residual life (MRL) plot was used to identify a threshold above which the generalized Pareto distribution (GPD) provides an adequate approximation to the tail behaviour of earthquake magnitudes. An approximately linear MRL curve indicates that the exceedances are consistent with the assumptions of the GPD model.

Figure 3 presents the empirical MRL plot for the Tehran Province earthquake catalogue. The plot exhibits an approximately linear pattern from a magnitude of 3.5 onward, indicating that the GPD provides an adequate model for exceedances above this threshold. Consequently, a threshold of $u = 3.5$ was selected for the subsequent analysis. This choice provides a reasonable balance between retaining a sufficient number of exceedances for reliable parameter estimation and satisfying the asymptotic assumptions underlying the POT methodology.

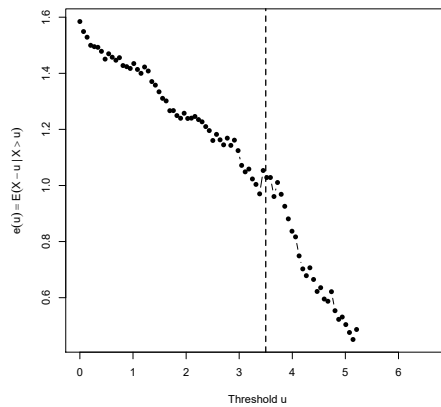


Figure 3: Mean residual life (MRL) plot for the earthquake magnitude data. The dashed line denotes the selected threshold ($u = 3.5$).

Using the selected threshold, earthquake magnitudes exceeding 3.5 were retained for fitting the generalized Pareto distribution. The adequacy of the fitted

GPD was subsequently assessed using probability (PP) and quantile (QQ) plots, as discussed below.

Following threshold selection, the generalized Pareto distribution was fitted to the exceedances using maximum likelihood estimation. The fitted model was evaluated using probability (PP) and quantile (QQ) plots, shown in Figure 4. In the PP plot, the empirical probabilities closely follow the reference line, while the QQ plot shows close agreement between the empirical and fitted quantiles. These diagnostic plots indicate that the generalized Pareto distribution provides a satisfactory fit to the exceedance data and supports the suitability of the selected threshold for modelling the upper tail of the earthquake magnitude distribution.

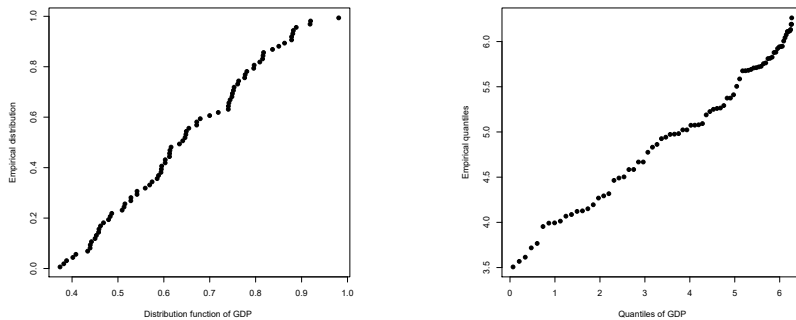


Figure 4: Diagnostic plots for the fitted generalized Pareto distribution: (left) probability (PP) plot and (right) quantile (QQ) plot.

Table 3 shows the descriptive statistics of magnitude (≥ 3.5) and elapsed time. Table 4 shows the sample correlation coefficients and corresponding p -values (for testing significant correlations) between magnitude (≥ 3.5) and elapsed time.

Table 3: Descriptive statistics of magnitude (≥ 3.5) and elapsed time.

Variable	Mean	SD	Min	Max	Skewness	Kurtosis
Magnitude (Richter)	3.99	0.48	3.5	6.3	0.73	4.7878
Elapsed time (hours)	1127.44	3625.79	0	25923.79	5.01	27.96

4.1.2 Spatial Trend Assessment and Marginal Modelling

Before fitting the copula, the marginal distributions were estimated, and a preliminary spatial analysis was conducted.

Figure 5 provides a visual assessment of the relationship between earthquake magnitude and epicentral latitude and longitude. No systematic spatial pattern is

Table 4: Sample correlation coefficient values and p -values.

Correlation Coefficient	Value	p -value
Pearson’s ρ	0.15	0.18
Kendall’s τ	0.07	0.55
Spearman’s ρ	0.07	0.53
Upper tail dependence λ_U	0.00	0.18
Lower tail dependence λ_L	0.00	0.11

apparent. This visual impression is confirmed by the regression analyses presented in Tables 5 and 6, where neither latitude nor longitude exhibits a statistically significant association with earthquake magnitude.

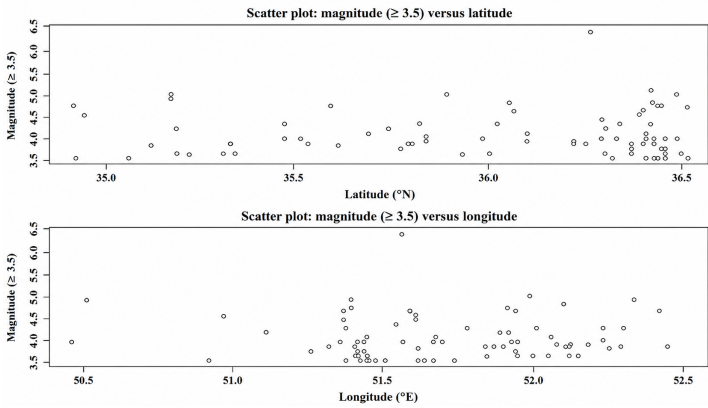


Figure 5: Scatterplots of earthquake magnitude ($M \geq 3.5$) versus epicentral latitude (top) and longitude (bottom) for the analysed earthquake catalogue. The corresponding linear regression results are reported in Tables 5 and 6.

Table 5: Linear regression of earthquake magnitude (≥ 3.5) on latitude.

Variable	Estimate	Std. Error	t -statistic	p -value
Intercept	8.00	2.52	0.755	0.754
Latitude	-0.07	0.22	-0.313	0.755
Overall model: $F = 0.098$, $p = 0.755$.				

Since each regression contains a single explanatory variable, the overall F -statistic is equal to the squared t -statistic for the slope coefficient. The resulting F -statistics are 0.098 for

latitude and 0.009 for longitude, with corresponding p -values of 0.755 and 0.926, respectively. Hence, the null hypothesis of no linear association between earthquake magnitude and either spatial variable cannot be rejected.

Table 6: Linear regression of earthquake magnitude (≥ 3.5) on longitude.

Variable	Estimate	Std. Error	<i>t</i> -statistic	<i>p</i> -value
Intercept	7.09	4.65	0.926	0.514
Longitude	-0.01	0.14	-0.093	0.926
Overall model: $F = 0.009$, $p = 0.926$.				

Because both regression models include an intercept, the algebraic sum of the ordinary least-squares residuals is zero up to numerical precision and therefore does not provide an additional goodness-of-fit measure.

Since magnitudes exceeding the threshold of 3.5 are analysed, the GPD provides a suitable model for the tail behaviour. Therefore, the GPD was fitted to the data and a goodness-of-fit test was performed. The results, presented in Table 7, indicate that the distribution fits the data well.

Table 7: Results of fitting GPD to magnitudes (≥ 3.5).

Variable	Shape	Scale	Test Statistic	<i>p</i> -value
Magnitude (> 3.5)	-1.75	10.97	0.96	0.52

For the elapsed time (hours) between two earthquakes with magnitudes greater than 3.5, several distributions—Exponential, Gamma, Lognormal, Weibull, and Lognormal-3 (three-parameter Lognormal)—were fitted. Among these, the Lognormal-3 distribution provided the best fit. Table 8 presents the results.

Table 8: Results of fitting Lognormal-3 to the elapsed time between two earthquakes.

Variable	Threshold	Meanlog	Sdlog	Test Statistic	<i>p</i> -value
elapsed time (hours)	0.07	3.27	3.47	0.966	0.32

4.1.3 Treatment of Temporal Dependence

For the Iranian Plateau, probabilistic modelling of aftershock occurrence and magnitude sequences has further emphasized the importance of accounting for temporal clustering before dependence analysis (Khansefid and Bakhshi, 2018).

So, the assumption that the pairs (M, T) are independent and identically distributed must be carefully justified. Before fitting the copula model, we assessed temporal structure using the ACF and Ljung–Box tests. For magnitudes exceeding 3.5, no significant serial correlation was detected (the corresponding *p*-value is 0.1389).

For the elapsed-time variable T associated with the retained above-threshold events, no statistically significant autocorrelation was detected either (the corresponding Ljung–Box p -value is 0.6726).

It should be emphasized that thresholding is not a formal declustering procedure and does not, by itself, make the retained earthquake events independent. Its primary role here is to restrict the analysis to the tail sample used in the peaks-over-threshold framework. Although excluding low-magnitude events may reduce the contribution of dense short-term aftershock sequences, larger events belonging to the same seismic sequence may still remain in the thresholded catalogue. Therefore, independence was not inferred from thresholding itself. Rather, residual temporal dependence was assessed separately using the ACF and Ljung–Box diagnostics reported above. The corresponding p -values provide no evidence of statistically significant serial autocorrelation at the examined lags, but this should not be interpreted as proof of complete independence or as a substitute for formal earthquake declustering.

4.2 Fitting Copula Function

The choice of candidate copula families was guided primarily by the empirical characteristics of the earthquake catalogue. As shown in Table 4, the estimated upper- and lower-tail dependence coefficients are both close to zero, indicating little evidence of tail dependence between earthquake magnitude and elapsed time. Consequently, copulas capable of representing weak dependence without imposing artificial tail behaviour were considered appropriate.

The selected copula families also provide complementary representations of dependence structures that are commonly encountered in practice. The Gaussian copula represents symmetric dependence without tail dependence and serves as a natural benchmark. The Frank and AMH copulas are flexible Archimedean families capable of modelling weak dependence while exhibiting no tail dependence, making them appropriate for datasets in which extreme co-movements are absent. The BB8 copula was included because, for suitable parameter values, it can also represent weak dependence with negligible tail dependence, while offering greater flexibility than the Gaussian and Archimedean copulas.

Copulas that impose substantial upper or lower tail dependence (e.g., BB1 or Clayton with large parameter values) were not considered further because the empirical tail-dependence coefficients reported in Table 4 are close to zero.

Because the Student- t copula is a natural elliptical alternative to the Gaussian copula, it was additionally examined as a robustness check. The likelihood increased as the degrees of freedom became large, with the numerical fit yielding an extremely large value ($\hat{\nu} \approx 4.68 \times 10^6$). For such a large number of degrees of free-

dom, the Student- t copula becomes very close to the Normal copula. Therefore, the Student- t fit did not provide a meaningfully different dependence structure from that represented by the Normal copula.

Collectively, these candidate models provide a diverse yet parsimonious set for evaluating model uncertainty under the dependence characteristics observed in the earthquake catalogue.

Because no single copula family can be expected to represent the dependence structure optimally in all situations, the selected candidate set is treated collectively within the Mahalanobis-distance-based model averaging framework rather than being ranked solely according to a single model selection criterion.

The model-averaging procedure consisted of bootstrap resampling, fitting each candidate copula to every resampled dataset, evaluating model stability using the Mahalanobis distance, and repeating this process over 1000 replications. The frequency with which each copula was selected was used to define model-averaging weights.

Table 9: Results of fitting copulas using the model-averaging approach.

Copula	p -value	Parameter	Kendall's τ	λ_L	λ_U	AIC	Contribution
Normal	0.352	$\hat{\theta} = 0.213$ (0.218)	0.136 (0.143)	0.000	0.000	-4.234	0.475
Frank	0.254	$\hat{\theta} = 0.596$ (0.207)	0.066 (0.023)	0.000	0.000	0.162	0.503
AMH	0.248	$\hat{\theta} = 0.325$ (0.347)	0.079 (0.078)	0.000	0.000	3.340	0.022
BB8	0.250	$\hat{\theta} = 1.001$ (0.049), $\hat{\delta} = 0.001$ (0.004)	0.000 (0.028)	0.000	0.000	4.002	0.000

The absence of statistically significant correlation does not preclude copula modelling. Rather, it motivates the use of a flexible dependence framework capable of detecting nonlinear relationships while explicitly accounting for model uncertainty.

Table 9 reports the model-averaging weights obtained from the bootstrap selection frequencies. The Normal and Frank copulas receive weights of 0.475 and 0.503, respectively, whereas the AMH copula contributes only 0.022 and the BB8 copula receives zero weight. These results indicate that the Normal and Frank copulas provide similarly adequate descriptions of the observed dependence structure, with neither model consistently outperforming the other across the bootstrap replications. Consequently, the data do not support selecting a single copula with high confidence, indicating the presence of residual model uncertainty.

Rather than relying on the copula with the largest weight, the proposed model-averaging procedure incorporates this uncertainty by combining the candidate models according to their empirical selection frequencies. Since the Gaussian and Frank copulas account for almost the entire model weight, the averaged copula

is primarily determined by these two families, while the small contribution of the AMH copula has only a minor influence on the resulting dependence model. Furthermore, the dominance of the Gaussian, Frank, and AMH copulas, all of which exhibit zero tail dependence, provides little empirical evidence for tail-dependent behaviour in the relationship between earthquake magnitude and elapsed time.

The weak empirical dependence reported in Table 4 and the dispersion of the model-averaging weights in Table 9 naturally raise the question of whether a dependence model is needed for this particular dataset. To address this issue directly, we additionally considered the independence copula, $C_{\perp}(u, v) = uv$, as a benchmark. Under independence, the conditional exceedance probability satisfies $P(U > u \mid V > v) = 1 - u$ and therefore does not vary with the elapsed-time threshold. The comparison with this benchmark, reported below in the conditional probability analysis, allows the practical contribution of copula modelling to be assessed directly rather than inferred solely from conventional correlation measures or model-selection weights.

Accordingly, the resulting averaged copula function is given by

$$\begin{aligned} C^*(u, v) = & 0.475 \Phi_{\rho=0.213}(\Phi^{-1}(u), \Phi^{-1}(v)) \\ & + 0.503 \left(-\frac{1}{0.596} \ln \left\{ 1 + \frac{(e^{-0.596u} - 1)(e^{-0.596v} - 1)}{e^{-0.596} - 1} \right\} \right) \\ & + 0.022 \frac{uv}{1 - 0.325(1 - u)(1 - v)}. \end{aligned} \quad (4.14)$$

4.3 Conditional Probability Analysis

After obtaining the copula function based on the model averaging method, as shown in equation 4.14, the conditional exceedance probability can be estimated. This conditional probability is:

$$P(M > m \mid T > t) = \frac{1 - F_M(m) - F_T(t) + C(F_M(m), F_T(t))}{1 - F_T(t)}. \quad (4.15)$$

Here, F_M and F_T are the marginal distributions of magnitude and elapsed time, respectively, and C is the chosen copula function. In our case, F_M is the GPD fitted to magnitudes above 3.5, F_T is the Lognormal-3 distribution fitted to elapsed times, and C is replaced by C^* from (4.14). Using (4.15), conditional probability curves can be plotted for selected values of t (e.g., $t = 100, 2000, 10000$ hours).

The elapsed time thresholds considered in this study (100, 2000, and 10000 hours) correspond approximately to 4, 83, and 417 days, respectively. These values represent short- to medium-term intervals within the observed catalogue and

are used to examine conditional behaviour rather than long-term recurrence probabilities.

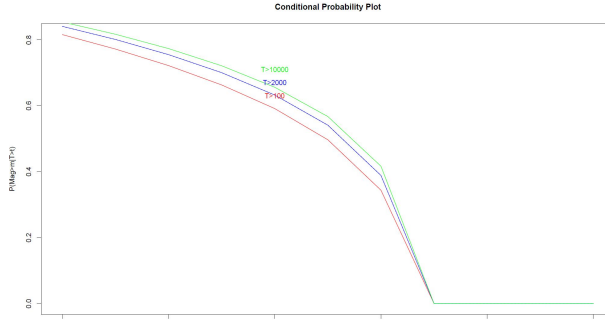


Figure 6: Estimated conditional exceedance probability $P(M > m | T > t)$ for elapsed time thresholds of 100, 2000, and 10000 hours.

As shown in Figure 6, the conditional exceedance curves have similar overall shapes, although a modest upward shift is observed as the elapsed-time threshold increases.

Figure 7 provides a direct comparison between the model-averaged copula and the independence benchmark. Under independence, the conditional exceedance probability is $1 - u$ and therefore does not vary with the elapsed-time threshold. The fitted model-averaged copula shows a measurable, although moderate, departure from this benchmark.

The difference becomes smaller toward the more extreme upper tail. At $u = 0.99$, the absolute differences from the independence benchmark are approximately 0.004, 0.008, and 0.010 for the same three elapsed-time thresholds. These results indicate that copula modelling does not reveal a strong dependence effect for this dataset, particularly in the extreme upper tail. Its role is instead to quantify the departure from independence and to show that the practical dependence effect is modest rather than to assume that a strong dependence structure is present.

5. Results and Discussion

The simulation study shows that the Mahalanobis-distance-based model averaging procedure can provide stable estimation across the dependence structures considered, although its accuracy depends on the underlying copula family and the strength of dependence. The model-averaged estimator generally exhibits lower variability than the corresponding single-model benchmark under the same repeated-sampling scheme, although the magnitude and direction of the bias de-

vs independence.pdf

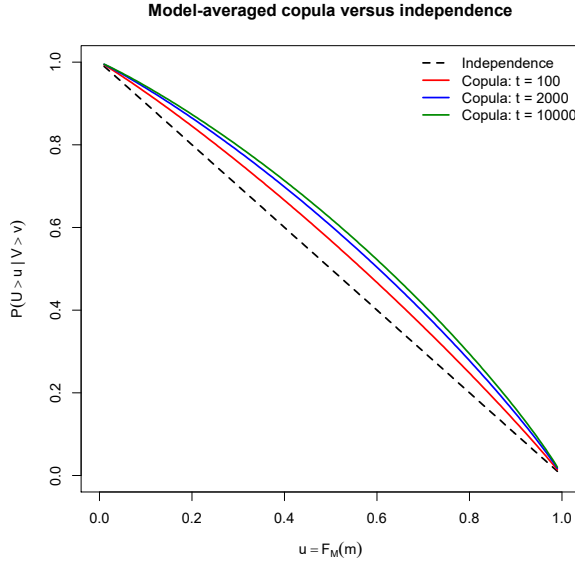


Figure 7: Comparison of conditional exceedance probabilities obtained from the model-averaged copula with the independence benchmark for elapsed-time thresholds $t = 100$, 2000, and 10000 hours.

pend on the underlying copula and the strength of dependence. These results support the use of selection-frequency-based weighting as a practical means of accounting for copula model uncertainty rather than relying exclusively on a single selected family.

The exploratory analysis supports the adopted marginal modelling of the earthquake catalogue. The MRL plot supports the selection of a threshold of $u = 3.5$ for the POT analysis, while the probability and quantile plots indicate that the generalized Pareto distribution provides an adequate fit to the exceedances. Similarly, the Lognormal-3 distribution satisfactorily models the elapsed times, providing suitable marginal distributions for the subsequent copula analysis.

The preliminary correlation analysis indicates no strong evidence of linear or monotonic association between earthquake magnitude and elapsed time (Table 4). Nevertheless, these measures provide only limited information about the under-

lying joint distribution and do not directly quantify the conditional exceedance probabilities that constitute the primary inferential objective of this study. The copula-based analysis therefore provides a framework for examining departures from independence, estimating conditional exceedance probabilities, and explicitly accounting for uncertainty among competing dependence models.

The conditional exceedance analysis provides a more nuanced interpretation of the practical role of dependence modelling for this catalogue. The conditional exceedance curves show only modest changes across the three elapsed-time thresholds considered. Moreover, direct comparison with the independence benchmark in Figure 7 shows that the model-averaged copula produces a measurable but moderate departure from independence over much of the probability range, with the difference becoming smaller toward the extreme upper tail. Thus, the results do not suggest that a strong dependence model is indispensable for this particular dataset. Rather, the copula analysis is useful for quantifying the extent of departure from independence and for showing that the practical dependence effect is limited, particularly for extreme exceedance probabilities.

This finding also indicates that elapsed time alone provides limited additional information about the magnitude of a subsequent earthquake in the Tehran Province catalogue. From a methodological perspective, the value of the proposed approach lies in allowing dependence inference and conditional exceedance assessment to incorporate uncertainty among competing copula models rather than conditioning the analysis on a single selected family.

Although the methodology is illustrated using earthquake data, it is not restricted to seismological applications. The proposed model averaging framework is applicable to a wide range of dependence modelling problems involving competing copula models, including applications in actuarial science, finance, environmental studies, and reliability analysis.

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Author Declarations

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Author Contributions. The author conceived the study, developed the

methodology, performed the analysis, interpreted the results, and prepared the manuscript.

Data Availability. The earthquake catalogue analysed during the current study was compiled from publicly available seismic records and processed by the author. The processed dataset is available from the corresponding author upon reasonable request.

Conflict of Interest. The author declares that there is no conflict of interest.

Research Involving Human Participants and/or Animals. This article does not involve human participants or animals.

Informed Consent. Informed consent was not required because this study did not involve human participants.

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